# Modelling Fiscal-Monetary Interaction and the Stability and Growth Pact in a Complex European Framework

## A New Approach with Differential Equations

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Abstract. The EMU involves many new interactions, since the establishment of the monetary union in 1999. That new framework induce a new research topic in economics. The main papers in the recent literature are based on a game theoretic framework like Barro/Grodon [2, 1]. Papers from Dixit and Lambertini (2003), Beetsma and Jensen (2003) and Beetsma and Bovenberg (1999) try to analyze that fiscal-monetary interaction relationship. Unfortunately there exist no dynamic interaction analysis which describes the complexity between Fiscal-Monetary and the Stability and Growth Pact. The taken approach in my paper with differential equations helps to understand the different institutional interaction-channels, trade-offs and check and balances. A surprising and new result is the trade-off between the breaching countries and the disciplined countries. Moreover a novelty in that framework is the proof that independent monetary policy is not sufficient to discipline fiscal policy free-rider behavior in a monetary union. The new insights improve the current modelling situation and helps to find some policy relevant conclusions for the reform process about the "European Stability and Growth Pact".

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## 1. Introduction

The huge reform discussion about the Stability and Growth Pact which emerge with the failing of the early warning in February 2002 und the failing to impose sanctions against Germany and France in November 2003, is a reason enough to analyze the current Stability Pact and the fiscal-monetary interaction framework in more detail. To find out the relevant trade-offs in the European fiscal-monetary interaction framework is a new research field for a short term. In a recent published book about 'Monetary and Fiscal Policies in EMU' Buti (2003, [11]) wrote:

> *Understanding the functioning of EMU and the interplay between monetary and fiscal authorities is and will remain a challenge for both academic research and policy-making for years to come. From their different perspectives, the contributions in this book provide the analytical instruments for undertaking a fascinating intellectual journey into the greatest monetary reform since Bretton Woods.*

It is therefore not surprising that there are relatively few models and theoretical arguments for the Stability and Growth Pact which was established in the subspace of fiscal-monetary interaction, since monetary union in 1999 (Dixit 2000,[15]). One of the most prominent results from the qualitative analysis of fiscal rules in pre-90th are: free-riding, moral hazard and asymmetric information are the challenges in a monetary union because of the new interactions. However, it is not known what is a *good* and *efficient* rule to manage fiscal-monetary interaction and there is no economic theory which explains the current 3% to GDP deficit threshold and the 60% debt threshold (De Grauwe 2003,[12]). Rather it seems to me non-trivial to analyze the European fiscal framework and especially the Stability and Growth Pact because it links on the one hand monetary and fiscal theory as well as incentive theory with institutional economic analysis (Brunila 2001,[9]). Both theory blocks are hardly linked because the last one have on the agenda to overcome the major drawbacks of the pure economic theory.

This paper focus on the existing pre-embryonic model frameworks and try to extend it to a more appropriate form for policy conclusions. Therefore you see immediately a new modelling approach that is based on a clear interdisciplinary approach between Economics and Mathematics.

When dealing with monetary-fiscal interaction in the monetary union it is a common practice (Dixit/Lambertini, Beetsma/Jensen, [14, 6, 3]) to study models which are based on Barro and Gordon (1983,[2]), Kydland and Perscott (1977,[21]) and thus on simple game theory. The lacks of all these models is the *no-dynamic* structure between the more complex framework of monetary-fiscal interaction and the Stability and Growth Pact. The construction of these models is based on the idea to model the Stability and Growth Pact as a fixed fine '$\psi$' for each additional unit of debt that is issued (cf. Beetsma and Uhlig, 1999, [5]). To use this modelling form is simple because at the moment there exists no real other option to make the Stability Pact also traceable in analytical models.

To extend the inside of the existing models and making the policy conclusions more relevant at most for the reform discussion about the Stability and Growth Pact, I build-up a new model approach based on *dynamic differential equations* (Hairer and Wanner 2002).

The remainder of this paper is structured as follows. Section 2 explains the main modelling idea. In section 3, I present a elementary-interaction model between fiscal policy and the Stability and Growth Pact. Section 4 extend the framework to a full interaction model between Fiscal-Monetary(-SGP) and the Stability and Growth Pact. After solving and analyzing the stability of the model I interpret the model results in section 5. Section 6 summarize the model conclusions und present some policy relevant modifications for the current reform discussion about the Stability and Growth Pact. Finally, Section 7 concludes the main body of the paper. All technicalities and proofs are relegated to an Appendix.

## 2. Model Framework

The model framework consists of three interacting institutions [5]. The first is the centralized monetary policy (European Central Bank, ECB). The primary objective of monetary policy is to maintain price-stability (art. 105 ECT). The monetary policy mainly interact with fiscal policies through the determination of price-level (cf FTPL) and interest rates. The second important institutional framework is the decentralized fiscal policy. The main difference between monetary and fiscal policy is that the fiscal policy retain a large degree of responsibility in the own national sovereignty. That imply three different interactions:

- **(i):** Fiscal policy interact with monetary policy. The budget decisions about deficit and debt have an impact on price-stability and thus on the monetary policy.
- **(ii):** Fiscal policy interact with the other fiscal policies in the monetary union because of the competition around the public good 'price-stability' provided by monetary policy. Thus the fiscal policy undertake free-rider behavior against the other participating member states within the monetary union. That free-riding incentives even increase in the framework of EMU (Beetsma/Bovenverg, [3]). To discipline the free-rider behavior in the European Monetary Union the so-called 'Stability and Growth Pact' was implemented.
- **(iii):** The Stability Pact is the third institution in the EMU. On the one hand it try to discipline fiscal policy and the free-rider behavior. On the other hand it helps the monetary policy to maintain the primary objective 'price-stability'.

The main task in the following paper is to analyze the interactions or interrelations in the European Monetary Union between these three institutional agents. I choice a dynamic concept that uses differential equations. The existing economic literature analyze fiscal-monetary interaction (Dixit and Lambertini 2003, Beetsma and Bovenberg 1999) in a game theoretic framework. The first approach to analyze also the Stability and Growth Pact (Beetsam and Uhlig 1999) again uses a game theoretic framework but without the real fiscal-monetary interaction structure. Moreover the economic approaches focuses more on monetary and real variables and their developments in the monetary union (Aarle et al. 2001,[25]).

But nobody try to analyze the institutional interaction in the European monetary Union and simultaneously using a dynamic framework.[2]

To illustrate the model framework graphically look to figure 1 in appendix B. The fiscal policy can influence the SGP and the monetary policy through to lax deficit and debt policy. The incentive to do this are: national interest, increase of re-election probability, national output stabilization, reaction to asymmetric and idiosyncratic shocks and the new free-riding behavior $\zeta$ which summarize all the incentives.

The next section try to model the interaction relationships between all three institutions with differential equations. The stringent modelling of that complex framework help us to learn something new about the interactions, impacts and causalities of the 'European Monetary Union'.

## 3. Basic Model

The following section describes the basic interaction model between the European fiscal policy and the Stability and Growth Pact. The primary target is to understand the evolution of breaching countries $'x(t)'$. Modelling the dynamics result in $(x(t) \geq 0)$:[3]

$$x' = (g - p * s)x, \qquad t > 0, \qquad x(0) = x_0 \tag{3.1}$$

with 'g' as the benefit through free-rider behavior of fiscal poliy in the European monetary union (Beetsma and Bovenverg 1999,[3])[4] and 's' represents disciplining sanctions from the 'Stability and Growth Pact'. The parameter 'p' is the probability of imposing sanctions. The intuition behind the first-order differential equation is:

- the increasing free-rider behavior '$g > 0$' in a monetary union increase with the number of countries that violate (against) the Stability and Growth Pact (SGP) because of the expected benefits.
- the sanction procedure '$s > 0$' in the Stability and Growth Pact try to reduce or discipline the free-rider behavior of national fiscal policies and thus reduce the number of breaching countries. But this mechanism works only efficient if the probability of imposing sanctions '$p > 0$', is sufficiently large.

The solution of that model is $x(t) = x_0 e^{(g-p*s)t}$. This imply an increasing number of breaching countries to the SGP, if free-riding incentives 'g' are larger than the disciplining sanctions 's'. In the current fiscal-SGP interaction system is the probability to impose sanctions very small.[5] That imply '$g > p*s$' and thus the number of breaching countries might be increasing.[6] But this modelling simplify

[2]An institutional analysis is done by Schmidt and Ohr 2003, [22].

[3]cf C. Schmeiser [23] and A. Jüngel (2003, [20]).

[4]cf Beetsma and Bovenberg (1999) show in the paper that free-riding behavior even increase in a monetary union.

[5]cf the failures of imposing early warnings against Germany, France (2002) and for example Italy (2004) and no sanctions against sinner states as Germany and France (2003) confirm that.

[6]That describes empirically the current situation in the EMU. The new breaching countries are Netherlands, United Kingdom, Greece and some of the new EAC.

the sanction mechanism and their impact on the national fiscal policy. A more realistic sanction mechanism looks like:

$$s = s(x) = s_0 + a * x, \qquad s_0, a \geq 0, \tag{3.2}$$

where $s_0$ represent the basic sanction amount and $a$ the marginal sanction rate or the idiosyncratic influence of the national fiscal policy. Substitute equation (3.2) in equation (3.1) yields:

$$\frac{dx}{dt} = x' = (\zeta^F - p * a * x)x, \qquad with \qquad t > 0, x(0) = x_0, \tag{3.3}$$

and $\zeta^F := g - p * s_0$. The differential equation above is a so-called logistic-differential equation (or 'Verhulst-Model'). The logistic modelling framework show also the sustainability of the number of breaching countries 'x(t)'. The solution of that differential equation through integration is:

$$\begin{aligned} t = \int_0^t = d\tau = \int_{x(0)}^{x(t)} \frac{dx}{(\zeta - pa * x)x} = \int_{x_0}^{x(t)} \frac{1}{\zeta}\left(\frac{1}{x} + \frac{pa}{\zeta - pa * x}\right) = \\ = \frac{1}{\zeta}\left(\ln\left[\frac{x(t)}{x_0}\right] + pa * \ln\left[\frac{\zeta - pa * x_0}{\zeta - pa * x(t)}\right]\right). \end{aligned} \tag{3.4}$$

To solve the last term to $x(t)$ result in:

$$x(t) = \frac{\zeta * x_0}{pa * x_0 + (\zeta - pa * x_0)e^{-\zeta t}}. \tag{3.5}$$

For $t \to \infty$:

$$x(t) \longrightarrow \begin{cases} \zeta/(p * a) & : \zeta > 0, \\ 0 & : \zeta < 0. \end{cases} \tag{3.6}$$

If the sanction mechanism is fully credible i.e. the implementation probability 'p' and sanction 's' are high, then the number of breaching countries converge to zero. But if free-riding behavior 'g' dominance the disciplining mechanism '(p*a)' then $\zeta > 0$ and thus the number of breaching countries convergence to '$\zeta/(p * a)$', a positive figure. The final number of violating countries increase with higher free-riding incentives and decrease if the sanctions are more credible and the economic impact of fiscal policy 'a' in the MU is relativ high.[7] The intuition behind the last term is that a higher influence of fiscal policy 'a' in MU imply normally a stronger sanction procedure (higher sanction amount or a punishment through the monetary policy) because of the increasing inflation danger. That might be a strong disciplining effect for member countries to reduce the fiscal policy below the 3% deficit and 60% debt thresholds. That finding suggest a sanction-threshold that should depend on the national GDP rate. The term $\zeta/(pa)$ could be interpreted as the natural intake capacity of breaching countries in a Monetary Union.

The next section extend the simple model with the monetary interaction level. Monetary policy interact both with fiscal policy and the Stability and Growth Pact.

[7] vice versa for a high policy impact of fiscal policy member states.

Take into consideration monetary policy help us to understand the full interaction framework.[8]

## 4. Full-interaction-model

Similar to the model description in section 2, I extend now the model with the monetary authority. Analyzing the compleat-complex system explains the current European fiscal-monetary interaction framework and the connection with the Stability and Growth Pact in a more realistic way as all the other existing economic models.

To model the evolution of monetary policy 'y(t)', I follow the similar approach with the differential equation above:

$$y' = (\zeta^M - d^{-1} * y)y, \qquad t > 0, \qquad y(0) = y_0 \tag{4.1}$$

where 'y' is monetary policy (for instance interest rates) and '$d \geq 0$' reflects the independence of monetary policy (or a weight i.e. the possibility to follow also other objectives as output stabilization). In the following section, I define $c := d^{-1}$. The intuition behind equation (4.1) is:

- if the free-riding behavior is dominant in the MU $\zeta^M > 0$ then monetary policy might punish fiscal policy additionally with higher interest rates.
- on the other hand if the monetary policy is fully independent ($d \to \infty$) then the primary objective 'price stability' ($\zeta^M$) has the whole weight. A more dependent Common Central Bank (CCB) ($d \to 0$) imply that output targets are more important. That have an explicit negative impact to interest rates (i.e. decline).

In a more realistic interaction framework, free-rider incentives '$\zeta$' depend also on the current number of breaching fiscal policy countries (Herzog 2004a,[17])[9]:

$$\zeta = \zeta^M(x) = -\zeta_1^M + \zeta_2^M x, \qquad with \qquad \zeta_1, \zeta_2 \geq 0 \tag{4.2}$$

where $\zeta_1$ represents disciplining incentives (for the number of non-breaching countries) and $\zeta_2$ describes the 'Cascade to the top' effect which was new explained in Herzog (2004a). Moreover the fiscal policy free-rider incentive $\zeta^F$ depend also on monetary policy:

$$\zeta^F(y) = \zeta_3^F - \zeta_4^F y, \qquad with \qquad \zeta_3, \zeta_4 \geq 0 \tag{4.3}$$

where $\zeta_3$ represents the increasing free-rider behavior in the Monetary Union (Beetsma and Bovenberg 1999) and $\zeta_4$ describes the 'Disciplining-Monetary-Policy' effect (interest rate effect).

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[8] cf because the independent European monetary policy can also discipline the fiscal policy for instance with higher interest rates.

[9] cf Fiscal Theory of Price Level, Woodford, 2003, [26].

Substituting equation (4.2) in equation (4.1) and also equation (4.3) in equation (3.1) yields the following system of differential equations. That system is very similar to the so-called 'Lotka-Volterra equations':[10]

$$
\begin{aligned}
x' &= (\zeta_3^F - \zeta_4^F * y - pa * x)x \qquad t > 0 \qquad x(0) = x_0 \\
y' &= (-\zeta_1^M + \zeta_2^M * x - c * y)y \qquad t > 0 \qquad y(0) = y_0
\end{aligned} \tag{4.4}
$$

To understand how the solution of the system evolves, I first simplify the system und assume $a = c = 0$.

$$
\begin{aligned}
x' &= (\zeta_3^F - \zeta_4^F * y)x \qquad t > 0 \qquad x(0) = x_0 \\
y' &= (-\zeta_1^M + \zeta_2^M * x)y \qquad t > 0 \qquad y(0) = y_0
\end{aligned} \tag{4.5}
$$

The differential equation system has two possible solutions $(x_1, y_1)'$ and $(x_2, y_2)'$:

$$
\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \textit{and} \qquad \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} \zeta_1^M / \zeta_2^M \\ \zeta_3^F / \zeta_4^F \end{pmatrix}
$$

To show the (asymptotic) stability or instability of the two solutions, I define the function $F(x, y)$ and calculate the eigenvalues of the system:

$$
F(x, y) = \begin{pmatrix} (\zeta_3^F - \zeta_4^F * y)x \\ (-\zeta_1^M + \zeta_2^M * x)y \end{pmatrix} \qquad x, y \geq 0, \tag{4.6}
$$

and the derivatives in the associated points are:

$$
F'(0,0) = \begin{pmatrix} \zeta_3^F & 0 \\ 0 & -\zeta_1^M \end{pmatrix}, \qquad \wedge \qquad F'\left(\frac{\zeta_1^M}{\zeta_2^M}, \frac{\zeta_3^F}{\zeta_4^F}\right) = \begin{pmatrix} 0 & -\zeta_4^F * \frac{\zeta_1^M}{\zeta_2^M} \\ \zeta_2^M * \frac{\zeta_3^F}{\zeta_4^F} & 0 \end{pmatrix},
$$

Now I calculate the eigenvalues of $F'(0,0)$:

$$
det|F'(0,0) - \lambda I| = -(\zeta_3^F - \lambda)(\zeta_1^M - \lambda) = 0, \tag{4.7}
$$

that imply $\lambda_1 = \zeta_3^F$ and $\lambda_2 = \zeta_1^M$. Because of the assumption that all $\zeta_i > 0$ $\forall i$, the two eigenvalues are positive. That imply an unstable equilibrium point $P_1(0,0)$.[11]

To determine the eigenvalue for $F'\left(\frac{\zeta_1^M}{\zeta_2^M}, \frac{\zeta_3^F}{\zeta_4^F}\right)$, I have to solve the following equation:

$$
det\left|F'\left(\frac{\zeta_1^M}{\zeta_2^M}, \frac{\zeta_3^F}{\zeta_4^F}\right) - \lambda I\right| = \lambda^2 + \zeta_2^M * \frac{\zeta_3^F}{\zeta_4^F} * \zeta_4^F * \frac{\zeta_1^M}{\zeta_2^M} = \lambda^2 + \zeta_3^F * \zeta_1^M = 0, \tag{4.8}
$$

[10]Goes back to Alfred James Lotka (1880 - 1949) and Vito Volterra (1860 - 1949).

[11]The instability can also be seen form the $det|F'(0,0)| < 0$.

the system is also instable if Re $\lambda_1 < 0$ and Re $\lambda_2 > 0$ (Strang 1988,[24]). That imply that the system is unstable around the second point $P_2(\zeta_1^M/\zeta_2^M; \zeta_3^F/\zeta_4^F)$. But from (4.8) follows directly that Im $\lambda_{1,2}$. The possibility of complex eigenvalues imply no real solution. To describe the solution behavior of the system near the point $(\zeta_1^M/\zeta_2^M; \zeta_3^F/\zeta_4^F)$ I rewrite the differential equation system (4.5) in the form:

$$\frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{x'}{y'} = \frac{(\zeta_3^F - \zeta_4^F * y)x}{(-\zeta_1^M + \zeta_2^M * x)y}, \tag{4.9}$$

and after integration I can rewrite the equation above as,

$$-ln[x^{\zeta_1}] + \zeta_2 * x = \int \frac{-\zeta_1 + \zeta_2 * x}{x} dx = \int \frac{\zeta_3 - \zeta_4 * y}{y} dy = ln[y^{\zeta_3}] - \zeta_4 * y - \alpha, \tag{4.10}$$

where $\alpha \epsilon \mathbb{R}$ is a integration constant. Thus *all* the solutions $(x(t), y(t))'$ satisfy the implicit solution:

$$ln[x(t)^{\zeta_1}] + ln[y(t)^{\zeta_3}] - \zeta_2 * x - \zeta_4 * y = \alpha \qquad \forall t \geq 0. \tag{4.11}$$

The integration constant $\alpha$ can be calculated from the initial condition $(x_0, y_0)$:

$$\alpha = ln[x_0^{\zeta_1}] + ln[y_0^{\zeta_3}] - \zeta_2 * x_0 - \zeta_4 * y_0. \tag{4.12}$$

I suggest that the solution set $(x(t), y(t))$ satisfy a closed-form solution in the environment $(\epsilon, \delta)$ around the point $(x_2, y_2)$:

$$x(t) = \frac{\zeta_1}{\zeta_2} + \epsilon * \sin[\omega t], \qquad \wedge \qquad y(t) = \frac{\zeta_3}{\zeta_4} + \delta \cos[\omega t], \tag{4.13}$$

with $\epsilon > 0$, $\delta \ll 1$ and $\omega > 0$. For $t = 0$ and after trivial aggregation it result:

$$\alpha = \zeta_1 \ln\left[\frac{\zeta_1}{\zeta_2}\right] + \zeta_3 \ln\left[\frac{\zeta_3}{\zeta_4}\right] - \zeta_1 - \zeta_3 + |O(\delta)| \qquad (\delta \longrightarrow 0). \tag{4.14}$$

The next step is now the approximation of the general solution $(x(t), y(t))$ (with second-order Taylor series) in the environment of $x_2 = \zeta_1/\zeta_2$ and $y_2 = \zeta_3/\zeta_4$ (cf Appendix A):

$$\zeta_1 * ln[x(t)] + \zeta_3 * ln[y(t)] - \zeta_2 * x(t) - \zeta_4 * y(t) \tag{4.15}$$

is equivalent to:

$$\begin{aligned} &= \zeta_1 \ln\left[\frac{\zeta_1}{\zeta_2}\right] + \zeta_2 \epsilon \sin \omega t + \zeta_3 \ln\left[\frac{\zeta_3}{\zeta_4}\right] + \zeta_4 \delta \cos \omega t + \frac{\zeta_2^2}{2\zeta_1}\epsilon^2 \sin^2 \omega t + \\ &\quad + \frac{\zeta_4^2}{2\zeta_3}\delta^2 \cos^2 \omega t - \zeta_1 - \zeta_2 \epsilon \sin \omega t - \zeta_3 - \zeta_4 \delta \omega t + O(\epsilon^3 + \delta^3), \end{aligned} \tag{4.16}$$

$$\begin{aligned} &= \zeta_1 \ln\left[\frac{\zeta_1}{\zeta_2}\right] + \zeta_3 \ln\left[\frac{\zeta_3}{\zeta_4}\right] - \zeta_1 - \zeta_3 + \frac{\zeta_2^2}{2\zeta_1}\epsilon^2 \sin^2 \omega t + \frac{\zeta_4^2}{2\zeta_3}\delta^2 \cos^2 \omega t + O(\epsilon^3 + \delta^3), \\ &= \alpha + O(\epsilon^2), \end{aligned}$$

if I choice

$$\frac{\zeta_2^2}{2\zeta_1}\epsilon^2 = \frac{\zeta_4^2}{2\zeta_3}\delta^2.$$

Thus you can conclude that my specified solution (4.13) solve the general system $(x(t), y(t))$ until a error term of order $O(\epsilon^2)$. Moreover we can see that the *Trajectories* $\{(x(t), y(t)) : t \geq 0\}$ are approximative ellipse around the point $(x_2, y_2)$ (cf figure 1 in the graphical appendix).

The intuition in short term: The simplified system-dynamics imply that the number of breaching countries increase so long as the monetary policy see no danger for price-stability in future. After the reaction of the monetary policy (increase in interest rates) the number of breaching countries decrease.

But the most interesting case is the general model (4.4) with $a \neq c \neq 0$. Now I calculate the general solution and proof the stability of the associated differential equation system. Out going from the solution, I analyze the interaction relationship between the three levels: Fiscal, Monetary and the Stability and Growth Pact.

The general model is described through the function $F(x, y)$:

$$F(x, y) = \begin{pmatrix} (\zeta_3^F - \zeta_4^F * y - pa * x)x \\ (-\zeta_1^M + \zeta_2^M * x - c * y)y \end{pmatrix} \qquad x, y \geq 0, \tag{4.17}$$

There are the following four solutions for the general model:

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \wedge \quad \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -\zeta_1/c \end{pmatrix} \quad \wedge \quad \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = \begin{pmatrix} \zeta_3/(pa) \\ 0 \end{pmatrix}$$

and $(x_4, y_4)'$ is the solution of the linear equation system:

$$\begin{pmatrix} pa & \zeta_4 \\ \zeta_2 & -c \end{pmatrix} \begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \begin{pmatrix} \zeta_3 \\ \zeta_4 \end{pmatrix}$$

using *Cramer's-rule* result in:

$$\begin{pmatrix} x_4 \\ y_4 \end{pmatrix} = \frac{1}{pac + \zeta_2 * \zeta_4} \begin{pmatrix} \zeta_3 * c + \zeta_4 * \zeta_1 \\ \zeta_3 * \zeta_2 - pa * \zeta_1 \end{pmatrix}$$

with $A := \zeta_3 * c + \zeta_4 * \zeta_1$ and $B := \zeta_3 * \zeta_2 - pa * \zeta_1$.

The second solution $(x_2, y_2)$ is a non possible stationary point because we have assumed $x, y \geq 0$. To find out the stability of the other three solutions, I derivate the function $F(x, y)$:

$$F'(x, y) = \begin{pmatrix} \zeta_3 - \zeta_4 - 2ax & -\zeta_4 x \\ \zeta_2 y & -\zeta_1 + \zeta_2 x - 2cy \end{pmatrix}. \tag{4.18}$$

Similar to the model in section 3 the point $(x_1, y_1)' = (0, 0)'$ is a non stationary solution because of $\mathrm{Re}\lambda > 0$.[12] The point $(x_3, y_3)$ is unstable, if $\zeta_1/\zeta_2 < \zeta_3/a$, and

[12]cf Heuser, H. (1986, p. 214,[18, 19]) because of Lipschitz-stetig (steady).

asymtotic stabil, if $\zeta_1/\zeta_2 > \zeta_3/a$. The point $(x_4, y_4)$ is positiv i.e. $x, y \geq 0$ for $\zeta_1/\zeta_2 < \zeta_3/a$. The eigenvalues from $F'(x_4, y_4)$ are:

$$\lambda_{1,2} = -\frac{1}{2}(aA + cB) \pm \sqrt{\frac{1}{4}(aA + cB)^2 - (\zeta_2\zeta_4 + ac)AB},$$

with $A, B > 0$. Because of $\mathrm{Re}\lambda_{1,2} < 0$ the point $(x_4, y_4)$ is asymptotic stable, i.e. $(x(t), y(t)) \longrightarrow (x_4, y_4)$ for $t \longrightarrow \infty$. That imply that the number of breaching countries convergence after a necessary time to:

$$x_4 = \frac{\zeta_3 c + \zeta_4\zeta_1}{pac + \zeta_4\zeta_3}. \tag{4.19}$$

The following equation system summarize the general model results:

(4.20)

$$\frac{\zeta_1}{\zeta_2} > \frac{\zeta_3}{pa} \quad : \quad x(t) \to \frac{\zeta_3}{pa} \quad and \quad y(t) \to 0 \quad for \quad t \to \infty;$$

$$\frac{\zeta_1}{\zeta_2} < \frac{\zeta_3}{pa} \quad : \quad x(t) \to \frac{\zeta_3 c + \zeta_4\zeta_1}{pac + \zeta_4\zeta_3} \quad and \quad y(t) \to \frac{\zeta_3\zeta_2 - pa\zeta_1}{pac + \zeta_3\zeta_4} \quad for \quad t \to \infty.$$

The interpretation of the results is delegated to the next section.

## 5. Interpretation of the Model Results

Now I discuss the mathematical analysis above a show some numerical simulations. Moreover that proofs the robustness and stability of my theoretical results, also in a more complex model framework.[13]

The first part of the general result is very similar to the findings in the basic model in section 3. But the implication from the constellation $\zeta_1/\zeta_2 > \zeta_3/(pa)$ is not realistic because of the monetary policy variable convergence to zero $y(t) \to 0$. Despite of that problem we show that even in that case the number of breaching countries convergence against a fixed ratio.

Assume that the free-rider incentives in a MU are small ($\zeta_3 \to 0$) and the number of disciplined member countries through the SGP are big ($\zeta_1 \to \infty$). Moreover assume that the ratio above exceeds the ratio of fiscal policy '$\zeta_3/(pa)$'. That constellation imply then the first proposition:

**Proposition 5.1.** *The number of breaching member states depends on real benefit from free-riding ($\zeta_3^F$) and the probability (credibility) of sanctions 'p' as well as the influence of the fiscal policy to the aggregate variables 'a'.*

**Proof 1.** *The proof follows directly from the first part of equation (4.20).*

*Remark* 1. You saw that a high sanction probability or a high influence of monetary variables (big countries) reduce the number of breaching countries. On the other hand the free-riding incentives induce the problem of lax fiscal policy behavior in

[13]cf figure 2 and 3 in the graphical appendix.

that framework. It is clear that the implementation of sanctions within the Stability and Growth Pact depends on the probability and credibility of the enforcement mechanism. On the one hand is the current sanction procedure within SGP worse because of the low probability and credibility in the enforcement. On the other hand is the partisan influence within the Ecofin-council and the pretty vage fiscal-institutional framework in the EMU a reason for the past failures in the SGP. That situation imply an increasing number of breaching countries despite of the fact that the good member countries dominante the EMU per definition.

The second part of the result is more interesting, because of the following more realistic cases:

**(a):** The impact of an individual country to monetary policy is relative small ($a \rightarrow 0$) and the sanction probability 'p' within the SGP is also low. Moreover the public good 'price-stability' induce a strong incentive to free-riding ($\zeta_3^F \rightarrow \infty$) (Beetsma and Bovenberg 2003, [4]). Thus the following ratio converge to infinity ($\zeta_3^F/(pa) \rightarrow \infty$). The other intended disciplining ratio $\zeta_1^M/\zeta_2^M$ is lower because of the weak fiscal consolidation effect in the MU '$\zeta_1$' and the new so-called 'Cascade-to-Top' effect within the interaction framework '$\zeta_2$'.

**(b):** Moreover in the observed case exists for both solution variables $(x(t), y(t))$ a stabil and strict positive outcome.

### 5.1. Now I discuss first the determinants of 'x(t)':

Is monetary policy sufficient to limit the number of breaching fiscal policy member states in the EMU? To answer that question, I find a really new trade-off for the monetary-fiscal interaction literature. The determinants of the breaching countries depend on the monetary independence variable 'c' and on the fiscal policy impact variable 'pa' to monetary policy.

**Proposition 5.2.** *The monetary policy independence 'c' and the fiscal policy impact to monetary policy 'p*a' can influence the number of breaching countries. Moreover monetary policy '$\zeta_4$' reduce the number of breaching countries but the consolidation incentives (good guys) from the SGP increase the number of breaching countries '$\zeta_1$'.*

**Proof 2.** *Derivation of x(t) is:* $\frac{\partial x(t)}{\partial c} = \frac{\zeta_4(\zeta_3^2 - pa\zeta_1)}{(pac+\zeta_3\zeta_4)^2} > 0$, *because of the assumption* $\zeta_3 \rightarrow \infty$ *and* $a \rightarrow 0$. *Increasing monetary independence ($c \downarrow$) imply a reduction of breaching countries. The impact of $\zeta_2$ is independent from the number of breaching countries. This is immediately clear from equation (4.19). But a higher impact of fiscal policy 'a' reduce the number of breaching countries in the interaction framework through a more restrictive monetary policy ($\zeta_4$). The disciplining effect through monetary policy is:*

$$\frac{\partial x(t)}{\partial \zeta_4} = \frac{c(\overbrace{p * a}^{\rightarrow 0}\zeta_1 - \overbrace{\zeta_3^2}^{\rightarrow\infty})}{(pac + \zeta_3\zeta_4)^2} < 0$$

.

*On the other hand imply the fiscal policy framework especially the Stability and Growth Pact '$\zeta_1$' an increasing number of undisciplined countries. See also equation (4.20)* □

*Remark* 2. A very interesting and new insight is the impact of $\zeta_1$. This variable describes the impact of the good guys (no-breaching countries) or the incentives of 'sound' and 'sustainable' fiscal policy. If the number of good guys increase in MU '$\zeta_1 \uparrow$' that imply a simultaneous increase of breaching countries 'x(t) $\uparrow$' because of the increasing free-riding incentives and the influence of the declining sanctions and their redistribution (graphical appendix: figure 1).

5.2. **Now I discuss in a second step the determinants of 'y(t)':**

Assume an initial constellation of parameters, where monetary policy can increase the interest rates. The following proposition shows that monetary policy is *very constraint* in the European Monetary Union to punish the breaching countries 'x(t)':

**Proposition 5.3.** *An increasing fiscal policy impact $\zeta_2$ increase monetary policy y(t) but restricted to: (i) Monetary impact in reducing free-riding incentives '$\zeta_4$' and (ii) the number of good guys i.e. the fiscal rules like the SGP '$\zeta_1$'.*

**Proof 3.** *The derivation is:* $\frac{\partial y(t)}{\partial \zeta_2} = \frac{\zeta_3}{(pac+\zeta_3\zeta_4)} > 0$ □

*Remark* 3. A really surprising finding in proposition 3 are the strong limits for monetary policy to discipline fiscal policy free-riding behavior in the European Monetary Union. If the number of disciplined member states (good guys) decreases or the fiscal framework has some weakens to discipline the free-riding behavior of fiscal policy then monetary policy fails or is unable to maintain 'price-stability'. The reason for that case are the limitations and constraints for fiscal-monetary policy interaction within a monetary union. The result show us again how important a good and efficient fiscal framework as the Stability and Growth Pact is. An strong and independent 'Common Central Bank' is not enough to solve the 'new' free-riding behavior in the European Monetary Union (graphical appendix figure 2 and 3).

The paradoxical situation with a full independent monetary policy but unable to discipline lax fiscal policy confirms the necessity of a strong and efficient fiscal-coordination framework in the European Monetary Union. Some modification proposals to the current SGP are in the next section.

Last but no least I discuss short the results for a theoretically compleat independent monetary policy. The result (4.20) changes to:

(5.1)
$$\frac{\zeta_1}{\zeta_2} < \frac{\zeta_3}{pa} \quad : \quad x(t) \to \frac{\zeta_1}{\zeta_3} \quad and \quad y(t) \to \frac{\zeta_3\zeta_2 - pa\zeta_1}{\zeta_3\zeta_4} \quad for \quad t \to \infty.$$

*Remark* 4. The last case illustrate that the number of breaching countries depend only on the impact of fiscal policy free-riding incentives $\zeta_3$ and the good guys (i.e.

fiscal policy rule; SGP) $\zeta_1$. If the number of disciplined countries (good guys) is large and/or if the fiscal rule is sufficiently strong then the number of breaching countries might be increasing. Moreover in the full independent case is the monetary policy more contractive but also more restricted. So a weak fiscal framework '$\zeta_1$' and a low impact to discipline fiscal policy '$\zeta_4$' are both big limits for monetary policy in reducing the free-riding incentives in the MU. This is a really surprising result and show the clear disadvantage of a full independent monetary policy within a monetary union framework and a weak Stability and Growth Pact. Additionally this finding suggest a clear benefit from more *fiscal policy coordination* because of the strong limits of monetary policy independence in the EMU.[14]

The next section discuss now some policy conclusions. I focus to the reform discussion about the Stability and Growth Pact and propose some modifications.

## 6. Policy conclusions for the SGP

When a father calls his baby ugly, people take notice and expect to find a seriously aesthetically challenged child. When the President of the European Commission calls the fiscal rules of the Stability and Growth Pact 'stupid' and 'rigid' it is clear that changes to the Pact are in the air (Buiter, 2003, [10]). In this sense I will establish here a 'New Reform' of the current Stability and Growth Pact. The reform suggestion consists of a detailed analysis of all existing reform proposals from European organizations or leading economists and the logic idea, which I have found in the analysis above and another paper (Herzog, 2004,[17]).

Apart from the economically absolutely desirable changes of the target variables and their application periods in the SGP (De Grauwe, 2003,[13]; Bofinger, 2003,[7]) the decision procedure in the Ecofin-council is certainly the most important one to modify. The main purpose is a good and efficient coordination mechanism that generate a sustainable fiscal policy framework in the EMU. Moreover all other changes to a more-dimensional 'target set' are pointless, provided that there aren't guaranteed adequate penetrations. Therefore, we need in Europe a more independent Ecofin-council to increase the probability in sanction enforcement and the implementation credibility of sustainable fiscal policy. To strengthen the European 'fiscal policy' and thus to generate an adequate opponent to monetary policy lies in the interest of the whole European society. The result is shown above because of the fact that monetary policy is unable to discipline fiscal policy in a monetary union. So I suggest a 'negative escape clause'. This has the following function: If the 'supranational' targets are excessive breached by a member state then the Ecofin council will pass the decision competence to the independent council. The disciplining mechanism is as described above no more monetary, which would be

[14] Consider the past situation in 2001 within the euro area and the case of the Irish economy particular. Growth in Ireland is currently running at 8.7% with unemployment below 5%, and its budget surplus largest within Europe. Inflation on the other hand has doubled to 4.6% in the last year - the highest rate in the EU. Interest rates set to promote growth in France and Germany are simply inappropriate for Ireland which is in danger of running out of control unless the economy can be slowed. Indeed Ireland's economy is small within the EU context. The implications for other periphery economies such as Spain, Italy, Portugal and Finnland seem clear because it erode its competitiveness while wages and price rise. That example show empirically the big limits of European monetary policy.

aggravate the situation, rather a equivalent punishment in the same amount but in a positive manner like obliged budget consolidations.[15]

A more modest solution for independence in the decision process can achieved with a 'Vote- and Reputation function' (cf. Herzog 2004, [16]). The idea is the following: Sanction decisions in the Ecofin council should crucially depend on the number of votes from the countries with prudent and sustainable fiscal policy. So the number of votes should correspond with its reputation in fiscal policy. A country with a prudent and sustainable fiscal policy structure should get more votes than unsustainable and breaching countries. I construct a 'reputation index' which depends on inflation, debt and deficit (perhaps growth) and calculate the amount of votes for each country. A country with prudent fiscal policy means - low inflation, low debt and deficit- gets more votes than a country with bad fiscal policy. This mechanism induce tow advantages: First it avoids policy dealing about votes (cf trading votes). Second it generates an intrinsically incentive through a market mechanism to more prudent and sustainable policy. Therefore, the Ecofin council and the national member states keeps her entire sovereignty (as long as they trade in compliance with the Pact's thresholds). The cost of breaching the Stability Pact are also very high but without aggravation of the economic and financial situation and with the advantage to make more credible and accountable decisions. Such a mechanism imply a more fitting opponent to the ECB and so no danger for the national financial bankruptcy.

Since 10 years, monetary theory is analyzed in economic literature we have not learned to transfer it to other topics. Now we should transfer such results also to fiscal policy (cf. Wypolsz, 2003) in Europe.

## 7. Conclusion

The new modelling approach of fiscal-monetary interaction with the Stability and Growth Pact through differential equations show some new and sometimes surprising results. The first new finding is that the system is stable with a positive number of breaching countries. The second and really astonishing result is that monetary policy is despite its independence restricted in punishing the breaching countries through higher interest rates. Moreover I find some new interaction channels for instance the positiv-relation between the disciplined member states (strong fiscal rules) and the number of breaching countries. All the findings invalidate the critics on the Stability and Growth Pact. Additionally I can show that the ECB is not able to discipline (eliminate) the free-riding behavior of the bad gays. This finding disprove nearly all critics which propose that influence channel. The policy conclusions out going from that modelling show us again how necessary and important an efficient fiscal framework like the Stability and Growth Pact in the European Monetary Union is.

[15]Moreover positive incentives (in my case, cuts) discipline fiscal policy more than monetary fees, because payments are arbitrary and recent empirical work confirms that supply-side consolidation (cuts) are stronger than demand-side effects like tax increases.

# 8. Appendix

## Appendix A. Taylor Series approximation

The second-order Taylor Series approximation for the both solutions (Bronstein 1997,[8]):

(A.1)
$$x(t) = \frac{\zeta_1}{\zeta_2} + \epsilon * \sin[\omega t], \qquad \wedge \qquad y(t) = \frac{\zeta_3}{\zeta_4} + \delta \cos[\omega t],$$

can be done for x(t) with,

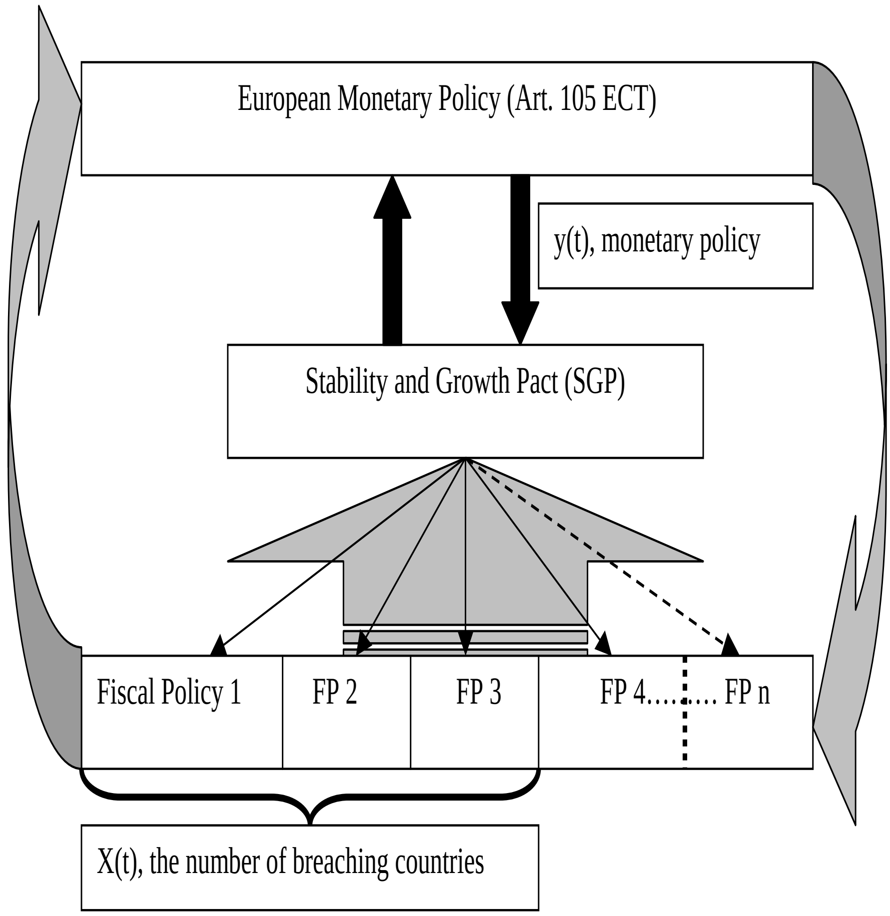
(A.2)
$$\ln x(t) = \ln\left[\frac{\zeta_1}{\zeta_2} + \epsilon * \sin[\omega t]\right] \approx \ln\left[\frac{\zeta_1}{\zeta_2}\right] + \frac{\zeta_2}{\zeta_1} * \epsilon * \sin[\omega t] - \frac{\epsilon^2 * \sin^2[\omega t]}{2} * \left(\frac{\zeta_2}{\zeta_1}\right)^2 + O(\epsilon^3),$$

and for y(t),

(A.3)
$$\ln y(t) = \ln\left[\frac{\zeta_3}{\zeta_4} + \delta * \cos[\omega t]\right] \approx \ln\left[\frac{\zeta_3}{\zeta_4}\right] + \frac{\zeta_4}{\zeta_3} * \delta * \sin[\omega t] - \frac{\delta^2 * \cos^2[\omega t]}{2} * \left(\frac{\zeta_2}{\zeta_1}\right)^2 + O(\delta^3).$$

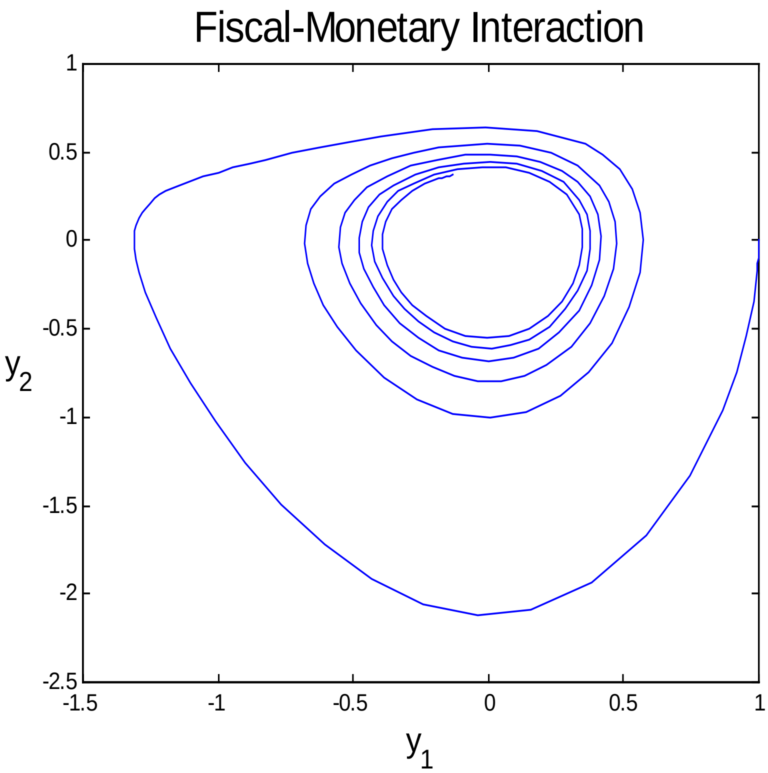
Substitute this two approximations in (4.15) and combine the terms. After that it result equation (4.16).

Appendix B. Model description



# Graphical Appendix

**Figure** 1:




Parameter specification:

$\zeta_1$= 0.6 ; $\zeta_2$= 0.05 ; $\zeta_3$= 1 ; $\zeta_4$= 1 ; p*a = 0.02, c = 0.5; $y_1$:= y; $y_2$:= x

**Matlab-programming (own-code):**

```
function yprime = fiscal(t,y)

yprime = [(0.6-0.05*y(1)-0.02*y(2))*y(2)
         (-1+y(2)-0.5*y(1))*y(1)];

tspan = [0 50];
yzero = [0 ;1];
[t,y] = ode45('fiscal',tspan,yzero);
plot(y(:,1),y(:,2))
xlabel('y_1','FontSize',14)
ylabel('y_2','FontSize',14,'Rotation',0,'HorizontalAlignment','right')
title('Fiscal-Monetary Interaction and some Paradox 2','FontSize',18)
```

**Figure** 2:

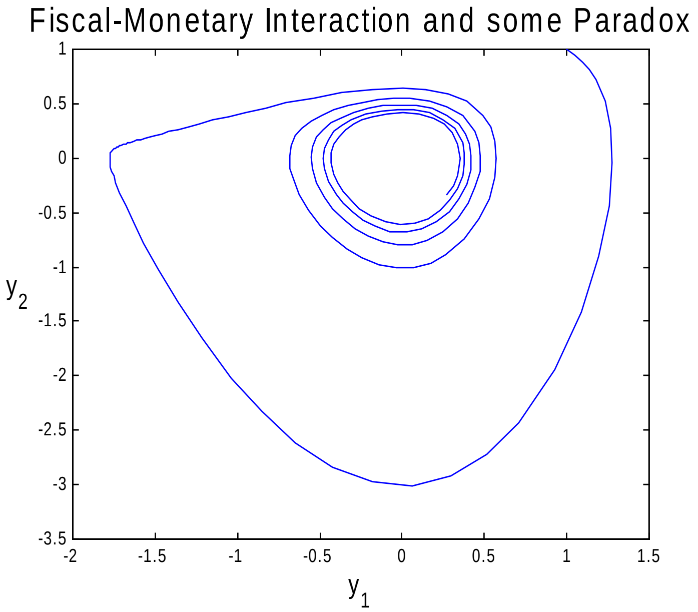
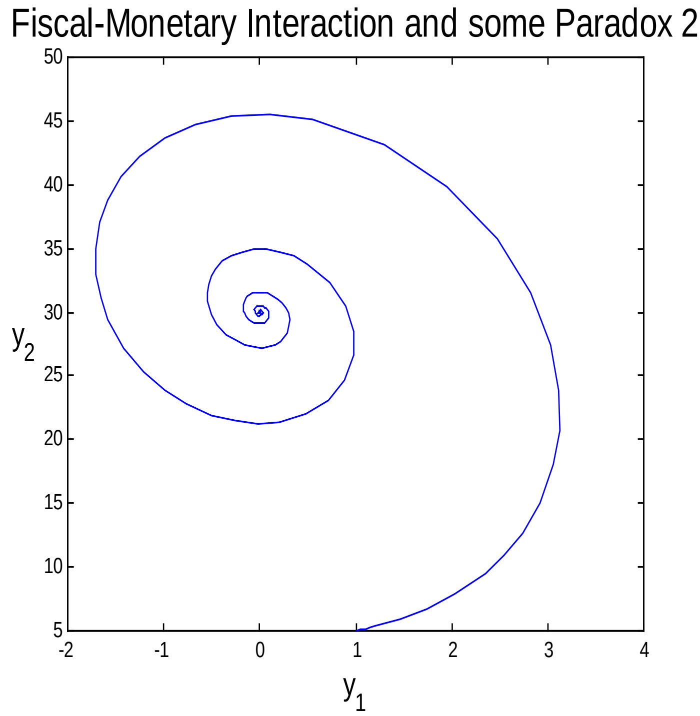



**Figure** 3:



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